

Model reference adaptive control with smooth switching scheme for piecewise linear systems and its application in turbofan engine control

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Abstract: A model reference adaptive control (MRAC) with smooth switching scheme was proposed for piecewise linear systems, and the method was utilized in turbofan engine control to avoid the discontinuity of control input. In this scheme, each sub-region of the operating envelope had its own MRAC controller, and smooth indicator function based smooth switching scheme was introduced to switch multiple controllers smoothly at the boundary of adjacent sub-regions. The Lyapunov stability analysis indicated that the proposed smooth switching scheme can guarantee the convergence of the closed-loop system during the controllers switching. The tracking error system was converted into a switched system to analyze the global stability of the closed-loop system. The advantage of the method was that the chattering of system output and instability caused by asynchronous switching can be eliminated. The simulation illustrates the effectiveness of the proposed control scheme in comparison with the existing MRAC controller with gain scheduling for turbofan engine.

Key words: model reference adaptive control; turbofan engine; smooth switching;
Lyapunov function; dwell time; Lyapunov stability

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With the increase in complexity and nonlinearity of aircraft engine, the control law that can ensure desired dynamic performance over a large operating envelope has received considerable attentions. One approach, favored by researchers in aircraft engine control, is to concentrate on the controller design of linear model which represent the dynamic performance of engine around a steady operating point, while the controller for operating conditions located between these neighboring points is determined by interpolating or switching between the corresponding

controllers^[1-6]. In the region around the selected operating point, stability and desired dynamic performance of closed-loop system can be provided by model reference adaptive control (MRAC) scheme.

Actually, many MRAC control laws for aircraft engine have been proposed in the Ref. [7-8]. Although these approaches can achieve desired control performance in a small range of operating conditions, MRAC scheme can't provide desired dynamic performance across a wide range of operating conditions^[9]. To overcome this

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problem, many improvements and results have been published^[10-12]. Among these control strategies, adaptive control with multiple models and switched system has received much attentions^[13-16]. In such scheme, the operating envelope is partitioned into a series of sub-regions, in which the dynamic behavior of aircraft engine is approximated by a linear model, and then controller is designed for each linear model. In principle, when the operating point moves from one sub-region to another, the controller or control decision changes synchronously^[17-18]. However, the MRAC controller switching from one to another may introduce undesired effect on the system. In addition, the switching signals available to the controllers are often later than the changing of system dynamics. That means, there exists asynchronous switching between the system dynamic and controller, and the asynchronous switching may lead to instability of the closed-loop system. Similar problems have been addressed previously in Ref. [16, 19], however, the control structures for sub-controller selection are complex.

The aim of this paper is to describe a simple smooth switching scheme, which is capable of switching one controller to another without discontinuity of control input in the entire range of operating conditions. The novelty of this work is that the smooth switching scheme combining the MRAC control method for linear system and the adaptive controllers that ensure the turbofan engine can operate throughout the flight envelope is designed. The adaptive laws for each sub-region can offset the bad effect of the changes in dynamic characteristics and ensure the major tracking behavior of the system. In this scheme, smooth indicator functions are introduced to make the controller evolve from one controller to another in a continuous manner.

This paper is organized as follows. In section 1, the piecewise linear system, state tracking control problem, and the smooth indicator functions for the smooth switching are introduced. In section 2, the controller structure is presented. Stability properties of the system is

studied in section 3. The application of the proposed scheme is demonstrated in section 4 followed by conclusion.

1 Problem statement

Generally, the nonlinear aero-thermodynamic model of aircraft engine can be described as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (1)$$

where $\mathbf{x} \in \mathbf{R}^n$ is the state vector, $\mathbf{u} \in \mathbf{R}^m$ is the control vector, and $\mathbf{f}(\cdot)$ is an n -dimensional nonlinear vector function which represents the engine dynamics.

Make $\Omega \in \mathbf{R}^{n+m}$ represents the state-input space which contains all possible system states and corresponding control inputs $(\mathbf{x}, \mathbf{u})$, and the space Ω can be partitioned into L polyhedral regions denoted by Ω_i , $i \in I = \{1, 2, \dots, L\}$, i. e. $\bigcup_{i=1}^L \Omega_i = \Omega$. The interior of Ω_i is denoted by Ω_i^0 , i. e. $\Omega_i \cap \Omega_j = \emptyset$, $\forall i \neq j, i, j \in I$. For convenience, the selected equilibrium point $(\mathbf{x}_{0,i}, \mathbf{u}_{0,i})$ for Ω_i locates in the center of the sub-region.

The nonlinear model (Eq. (1)) can be approximated by a finite number of terms of its Taylor series. If ignoring the high order terms, the linearization approximation model of the aircraft engine can be expressed as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \approx \mathbf{f}(\mathbf{x}_{i,0}, \mathbf{u}_{i,0}) + \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{(\mathbf{x}_{i,0}, \mathbf{u}_{i,0})} \cdot \Delta \mathbf{x} + \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{(\mathbf{x}_{i,0}, \mathbf{u}_{i,0})} \cdot \Delta \mathbf{u} \quad (2)$$

$$\Delta \dot{\mathbf{x}} = \mathbf{A}(t) \Delta \mathbf{x} + \mathbf{B}(t) \Delta \mathbf{u}$$

where $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}_{i,0}$, $\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}_{i,0}$, $\mathbf{A} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{(\mathbf{x}_{i,0}, \mathbf{u}_{i,0})}$ and $\mathbf{B} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{(\mathbf{x}_{i,0}, \mathbf{u}_{i,0})}$ are the parameter matrices of the linearized system. For the convenience of formula derivation, $\Delta \mathbf{x}$, $\Delta \mathbf{u}$ are denoted by $\mathbf{x}$, $\mathbf{u}$, and there is

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} \quad (3)$$

That means, in the sub-region Ω_i , the nonlinear system Eq. (1) can be approximated by the linear system $\dot{\mathbf{x}}_i = \mathbf{A}_i \cdot \mathbf{x}_i + \mathbf{B}_i \cdot \mathbf{u}_i$. The current state-input vector determines which sub-system is activated. The parameters discontinuities of $(\mathbf{A}(t), \mathbf{B}(t))$ can be characterize by indicator function^[20]

$$\chi_i(t) = \begin{cases} 1, & (\mathbf{x}, \mathbf{u}) \in \Omega_i \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

The common boundary of two adjacent sub-regions is merged into one of them, i. e. $\sum_{i=1}^L \chi_i = 1$. When the state of system (Eq. (1)) enters into the sub-region Ω_i , the i -th subsystem ($\mathbf{A}_i, \mathbf{B}_i$) would be active and $\chi_i(t) = 1$. Then, the controlled sub-system can be expressed as

$$\dot{\mathbf{x}} = \sum_{i=1}^L \mathbf{A}_i \cdot \chi_i(t) \cdot \mathbf{x} + \sum_{i=1}^L \mathbf{B}_i \cdot \chi_i(t) \cdot \mathbf{u}_i \quad (5)$$

During the system operation, corresponding to the indicator function $\chi_i(t), i \in I$, there is the switching sequence $\{(l_0, t_0), \dots, (l_j, t_j), (l_i, t_i), \dots, l_k \in I, 0 < t_k \in \mathbf{R}, 0 < k \in \mathbf{Z}\}$, in which l_k means that the state of system (Eq. (1)) enters the sub-region Ω_{l_k} and l_k -th subsystem is activated, and t_k is the instant of time that the state of system (Eq. (1)) enters the sub-region Ω_{l_k} and the active subsystem is switched to the l_k -th one.

MRAC method is used to design the controller of each sub-system. In addition, the smooth indicator functions (Eq. (6)) is introduced to serve as smooth switching scheme for controllers switching. Fig. 1 depicts the curve of the smooth indicator functions.

$$\begin{aligned} \gamma_j(\tau_i(t)) &= \frac{e^{\tau_1(t)}}{e^{\tau_1(t)} + e^{-c_4 + k_3 \cdot \tau_1(t)}} \\ \gamma_i(\tau_i(t)) &= \frac{e^{-c_4 + k_3 \cdot \tau_1(t)}}{e^{\tau_1(t)} + e^{-c_4 + k_3 \cdot \tau_1(t)}} \end{aligned} \quad (6)$$

where $i = j + 1$ and $i, j \in I$, $\tau_i(t)$ denotes the length of time to complete the controller switching. When $t \in [t_i, t_i + \tau(t_i))$, there is $\chi_i(t) = 1$,

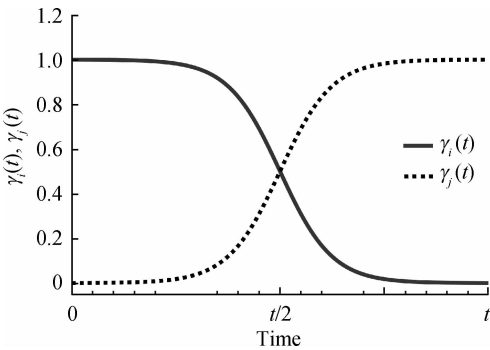


Fig. 1 Smooth indicator function

$\gamma_i(t) \in [0, 1], \gamma_j(t) \in [0, 1]$. The control input during the switching process is determined by $\mathbf{u}_{ji}(t) = \gamma_j(t) \cdot \mathbf{u}_j + \gamma_i(t) \cdot \mathbf{u}_i$, where $\gamma_j(t)$ and $\gamma_i(t)$ are monotonically increasing and monotonically decreasing functions, respectively.

2 MRAC controller design

In each sub-region, a reference linear system is selected to specify the desired response for the controlled plant. System state $\mathbf{x}(t)$ is forced to track the reference trajectory $\mathbf{x}_m(t)$ generated by the reference model. The piecewise linear reference system is given by

$$\dot{\mathbf{x}}_m = \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{r}_i \quad (7)$$

where $\mathbf{A}_m = \sum_{i=1}^L \mathbf{A}_{m,i} \cdot \chi_i(t), \mathbf{B}_m = \sum_{i=1}^L \mathbf{B}_{m,i} \cdot \chi_i(t), \mathbf{x}_{m,i} \in \mathbf{R}^n$ and $\mathbf{r}_i \in \mathbf{R}^p$. Suppose that $\mathbf{A}_{m,i}, i \in I$ is stable and there exists symmetric, positive definite matrix $\mathbf{P}_i \in \mathbf{R}^{n \times n}$ and positive number λ_i such that

$$\mathbf{A}_{m,i}^T \mathbf{P}_i + \mathbf{P}_i \mathbf{A}_{m,i} \leq -\lambda_i \mathbf{P}_i \quad i \in I \quad (8)$$

The exponential stability of Eq. (7) can be demonstrated by studying the exponential stability of its homogeneous system^[20]. If the current system state belongs to Ω_i , the reference system ($\mathbf{A}_{m,i}, \mathbf{B}_{m,i}$) is activated and $\chi_i(t) = 1$.

The following controller structure is applied

$$\mathbf{u}(t) = \mathbf{K}_x^T \mathbf{x} + \mathbf{K}_r \mathbf{r} \quad (9)$$

where $\mathbf{K}_x(t) = \sum_{i=1}^L \mathbf{K}_{x,i}(t) \cdot \chi_i(t), \mathbf{K}_r(t) = \sum_{i=1}^L \mathbf{K}_{r,i}(t) \cdot \chi_i(t)$ are the estimates of nominal controller parameters $\mathbf{K}_x^*$ and $\mathbf{K}_r^*$, respectively. The $\mathbf{K}_{x,i}^*, \mathbf{K}_{r,i}^*$ fulfill the following matching conditions

$$\mathbf{A}_{m,i} = \mathbf{A}_i + \mathbf{B}_i \mathbf{K}_{x,i}^T, \mathbf{B}_{m,i} = \mathbf{B}_i \mathbf{K}_{r,i}^* \quad (10)$$

For the nominal control gains, there are the following assumptions:

Assumption 1 The estimated controller parameters $\mathbf{K}_{x,i}, \mathbf{K}_{r,i}, i \in I$ are bounded, i. e. $\mathbf{K}_{x,i}^T \in [\underline{\mathbf{K}}_{x,i}, \bar{\mathbf{K}}_{x,i}], \mathbf{K}_{r,i} \in [\underline{\mathbf{K}}_{r,i}, \bar{\mathbf{K}}_{r,i}]$.

Assumption 2 There exists constant matrices $\mathbf{S}_i \in \mathbf{R}^{p \times p}$ such that

$$\mathbf{K}_{r,i}^* \mathbf{S}_i = \mathbf{S}_i^T \mathbf{K}_{r,i}^{*T} > \mathbf{0}$$

When $(\mathbf{x}, \mathbf{u}) \in \Omega_i$, substituting Eq. (9) into

Eq. (3), then the closed-loop can be written as

$$\dot{\mathbf{x}}_i(t) = \mathbf{A}_{m,i}\mathbf{x}_i + \mathbf{B}_{m,i}\mathbf{r}_i + \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{x,i}^T\mathbf{x}_i + \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{r,i}\mathbf{r}_i \quad (11)$$

Following from Eq. (11) and Eq. (7), the tracking error dynamic of the closed-loop system can be written as

$$\dot{\mathbf{e}}_i = \mathbf{A}_{m,i}\mathbf{e}_i + \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}[\tilde{\mathbf{K}}_{x,i}^T \quad \tilde{\mathbf{K}}_{r,i}] \begin{bmatrix} \mathbf{x}_i \\ \mathbf{r}_i \end{bmatrix} \quad (12)$$

Where $\mathbf{e}_i = \mathbf{x}_i - \mathbf{x}_{m,i}$, $\tilde{\mathbf{K}}_{x,i}^T = \mathbf{K}_{x,i}^T - \mathbf{K}_{x,i}^{*T}$ and $\tilde{\mathbf{K}}_{r,i}^T = \mathbf{K}_{r,i}^T - \mathbf{K}_{r,i}^{*T}$. Due to the variation of the plant model $(\mathbf{A}, \mathbf{B})$, the estimated control parameters are updated to ensure the asymptotic convergence of the tracking error. Both these two parameters are adjusted adaptively as

$$\begin{aligned} \dot{\mathbf{K}}_{x,i}^T(t) &= -\mathbf{F}_{x,i}\mathbf{S}_i^T\mathbf{B}_{m,i}^T\mathbf{P}_i\mathbf{e}_i\mathbf{x}_i^T + \mathbf{F}_{x,i}^T(t) \\ \dot{\mathbf{K}}_{r,i}^T(t) &= -\mathbf{F}_{r,i}\mathbf{S}_i^T\mathbf{B}_{m,i}^T\mathbf{P}_i\mathbf{e}_i\mathbf{r}_i^T + \mathbf{F}_{r,i}^T(t) \end{aligned} \quad (13)$$

where $\mathbf{F}_{x,i}, \mathbf{F}_{r,i}$ are positive scalar matrices given by designers, and $\mathbf{F}_{x,i}^T(t), \mathbf{F}_{r,i}^T(t)$ are defined to confine the estimated parameters inside the given bound.

3 Stability and tracking properties

Switching generally affects the smoothness of dynamical systems' outputs and complicates the stability analysis. When $t \in [t_{j-1}, t_j)$, $\chi_j(t) = 1, \chi_i(t) = 0$, i. e. the j -th sub-system is activated. When $t \in [t_j, t_i)$, $\chi_j(t) = 0, \chi_i(t) = 1$, i. e. the i -th subsystem is activated. The smooth indicator functions (Eq. (6)) are introduced to achieve smooth switching between the adjacent subsystems $(\mathbf{A}_i, \mathbf{B}_i)$ and $(\mathbf{A}_j, \mathbf{B}_j)$. When $t \in [t_j, t_j + \tau(t_i)), \tau(t_i) \in \mathbf{R}^+, t_j < t_j + \tau(t_i) < t_i$ the control law takes the form

$$\mathbf{u}_{j,i} = \gamma_j(t)\mathbf{u}_j^i + \gamma_i(t)\mathbf{u}_i \quad (14)$$

With the terms $\mathbf{u}_j^i = \mathbf{K}_{x,j}^T\mathbf{x}_j + \mathbf{K}_{r,j}\mathbf{r}_j$, $\mathbf{u}_i$ is the control input generated by the j -th controller. The parameters $\mathbf{K}_{x,j}^T$ and $\mathbf{K}_{r,j}$, reference signal $\mathbf{r}_j$, system state $\mathbf{x}_j$ and tracking error $\mathbf{e}_j$ are defined in Ω_j . That means, the j -th controller is used to control the i -th sub-system partly until the smooth switching process is completed. Consider the state and variable transformation given by $\mathbf{x}_j = \mathbf{x}_i + \mathbf{c}_{x,ji}$, $\mathbf{r}_j = \mathbf{r}_i + \mathbf{c}_{r,ji}$ and $\mathbf{e}_j = \mathbf{e}_i + \mathbf{c}_{x,ji} + \mathbf{c}_{m,x,ji}$, where $\mathbf{c}_{x,ji} \in \mathbf{R}^n$, $\mathbf{c}_{r,ji} \in \mathbf{R}^p$, and $\mathbf{c}_{x,ji}, \mathbf{c}_{m,x,ji} \in \mathbf{R}^n$, these transformations are defined in

region $\Omega_{j,i} \in \Omega_i$.

Consider the parameters $\mathbf{x}_i, \mathbf{r}_i, \mathbf{e}_i$ in Ω_i , and assuming that the required parameters for the calculation of $\mathbf{u}_j$ can be transformed from the corresponding parameters in Ω_j , the same control architecture and control law yield the $\mathbf{u}_{j,i}$. Then Eq. (14) can be expressed as

$$\mathbf{u}_{j,i} = \mathbf{K}_{x,j}^T\mathbf{x}_i + \mathbf{K}_{r,j}\mathbf{r}_i + \gamma_j(\mathbf{K}_{x,j}^T\mathbf{c}_{x,ji} + \mathbf{K}_{r,j}\mathbf{c}_{r,ji} + \mathbf{c}_{u,ji}) \quad (15)$$

where $\mathbf{K}_{x,j}^T = \gamma_j\mathbf{K}_{x,j}^T + \gamma_i\mathbf{K}_{x,i}^T$, $\mathbf{K}_{r,j} = \gamma_j\mathbf{K}_{r,j} + \gamma_i\mathbf{K}_{r,i}$.

Substituting Eq. (15) into system Eq. (5), there is

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_{m,i}\mathbf{x}_i + \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1} \cdot (\mathbf{K}_{x,ji}^T - \mathbf{K}_{x,i}^{*T})\mathbf{x}_i + \\ &\quad \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\mathbf{K}_{r,ji}\mathbf{r}_i + \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1} \\ &\quad (\mathbf{K}_{x,j}^T\mathbf{c}_{x,ji} + \mathbf{K}_{r,j}\mathbf{c}_{r,ji} + \mathbf{c}_{u,ji}) \end{aligned} \quad (16)$$

With substitution of (16) Eq. into Eq. (7), there is

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{A}_{m,i}\mathbf{e}_i + \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{x,j}^T\mathbf{x}_i + \\ &\quad \gamma_i\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{x,i}^T\mathbf{x}_i + \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}(\mathbf{K}_{x,j}^{*T} - \mathbf{K}_{x,i}^{*T})\mathbf{x}_i + \\ &\quad \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{r,j}\mathbf{r}_i + \gamma_i\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}\tilde{\mathbf{K}}_{r,i}\mathbf{r}_i + \\ &\quad \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}(\mathbf{K}_{x,j}^* - \mathbf{K}_{x,i}^*)\mathbf{r}_i + \\ &\quad \gamma_j\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}(\mathbf{K}_{x,j}^T\mathbf{c}_{x,ji} + \mathbf{K}_{r,j}\mathbf{c}_{r,ji} + \mathbf{c}_{u,ji}) \end{aligned} \quad (17)$$

Lemma^[20] Considering the tracking error dynamic Eq. (15) and Eq. (17). For any $R_3 > 0$, there exists a set which satisfies $S(R_3) = \{\mathbf{e}_i \mid \|\mathbf{e}_i(t)\| \leq R_3\}$. If $\mathbf{e}_i(t) \in \mathbf{R}^n/S(R_3)$, then there exists $\mu_e > 0$ such that

$$\begin{aligned} V_{j,i}(t) &\leq \mu_e \|\mathbf{e}_i(t)\|^2 \\ V_i(t) &\leq \mu_e \|\mathbf{e}_i(t)\|^2 \end{aligned} \quad (18)$$

Theorem 1 When $t \in [t_i, t_i + \tau(t_i)), i = j + 1, i, j \in I, \chi_j(t) = 0, \chi_i(t) = 1$, with the help of smooth switching scheme (Eq. (6)), the controller for sub-system $(\mathbf{A}_i, \mathbf{B}_i)$ is switched as the active controller gradually. Considering the closed-loop system consisting of the piecewise linear model (Eq. (5)), the reference model (Eq. (7)), the smooth switching scheme (Eq. (6)) and the controller (Eq. (14)) adjusted as Eq. (13). For any initial tracking error $\|\mathbf{e}_i(t_i)\| < \delta$, $\delta \in \mathbf{R}^+$, there exists $k_3 \in \mathbf{R}^+$ which satisfies $1/\rho \cdot R_2(a_{1,ji} + a_{3,ji} + a_{2,ji})\|\mathbf{e}_i\|^2 + \alpha_j \leq k_3$ to guarantee the convergence of $\mathbf{e}_i(t), \tilde{\mathbf{K}}_{x,i}^T$ and $\tilde{\mathbf{K}}_{r,i}$,

and all signals in the closed-loop system are bounded during the controller switching process. where $\alpha_j = \frac{1}{\rho} a_{1,j} - 1$.

Proof When $t \in [t_i, t_i + \tau(t_i))$, we have $\chi_i(t) = 1, \gamma_j(t) \in [0, 1], \gamma_i(t) \in [0, 1]$, and the current system state belongs to the sub-region $\Omega_i(t)$. Consider the Lyapunov function

$$V_{j,i}(t) = \mathbf{e}_i^T \mathbf{P}_i \mathbf{e}_i + \gamma_j [\text{tr}(\tilde{\mathbf{K}}_{x,j} \mathbf{M}_{s,j}^{-1} \tilde{\mathbf{K}}_{x,j}^T) + \text{tr}(\tilde{\mathbf{K}}_{r,j}^T \mathbf{M}_{s,j}^{-1} \tilde{\mathbf{K}}_{r,j})] + \gamma_i [\text{tr}(\tilde{\mathbf{K}}_{x,i} \mathbf{M}_{s,i}^{-1} \tilde{\mathbf{K}}_{x,i}^T(t)) + \text{tr}(\tilde{\mathbf{K}}_{r,i}^T \mathbf{M}_{s,i}^{-1} \tilde{\mathbf{K}}_{r,i})] \quad (19)$$

Under assumption 1, the following equality hold

$$\begin{aligned} \dot{V}_{j,i}(t) &= \dot{\mathbf{e}}^T \mathbf{P}_i \mathbf{e} + \mathbf{e}^T \mathbf{P}_i \dot{\mathbf{e}} + 2 \sum_{k=1}^2 \gamma_k \text{tr}(\tilde{\mathbf{K}}_{x,k} \mathbf{M}_{s,k}^{-1} \dot{\tilde{\mathbf{K}}}_{x,k}^T(t)) + 2 \sum_{k=1}^2 \gamma_k \text{tr}(\tilde{\mathbf{K}}_{r,k} \mathbf{M}_{s,k}^{-1} \dot{\tilde{\mathbf{K}}}_{r,k}^T) + \\ &2 \sum_{k=1}^2 \dot{\gamma}_k(t) \text{tr}(\tilde{\mathbf{K}}_{x,k} \mathbf{M}_{s,k}^{-1} \tilde{\mathbf{K}}_{x,k}^T) + 2 \sum_{k=1}^2 \dot{\gamma}_k(t) \text{tr}(\tilde{\mathbf{K}}_{r,k} \mathbf{M}_{s,k}^{-1} \tilde{\mathbf{K}}_{r,k}^T) \end{aligned} \quad (22)$$

With the adaptive law Eq. (13) and the assumption 2, there are

$$\begin{aligned} \text{tr}(\tilde{\mathbf{K}}_{x,i} \mathbf{M}_{s,i}^{-1} \dot{\tilde{\mathbf{K}}}_{x,i}^T) &= -\mathbf{e}_i^T \mathbf{P}_i \mathbf{B}_{m,i} \mathbf{K}_{r,i}^{*-1} \tilde{\mathbf{K}}_{r,i}^T \mathbf{x}_i + \text{tr}(\tilde{\mathbf{K}}_{x,i} \mathbf{M}_{s,i}^{-1} \mathbf{F}_{x,i}^T) \\ \text{tr}(\tilde{\mathbf{K}}_{r,i}^T \mathbf{M}_{s,i}^{-1} \dot{\tilde{\mathbf{K}}}_{r,i}) &= -\mathbf{e}_i^T \mathbf{P}_i \mathbf{B}_{m,i} \mathbf{K}_{r,i}^{*-1} \tilde{\mathbf{K}}_{r,i}^T \mathbf{r}_i + \text{tr}(\tilde{\mathbf{K}}_{r,i}^T \mathbf{M}_{s,i}^{-1} \mathbf{F}_{r,i}^T) \end{aligned} \quad (23)$$

Combining Eq. (17), Eq. (22) and Eq. (23), the derivative of $V_{j,i}(t)$ can be expressed as

$$\begin{aligned} \dot{V}_{j,i}(t) &\leq \mathbf{e}_i^T \lambda_i \mathbf{P}_{j,i} \mathbf{e}_i + 2\gamma_j \mathbf{e}_i^T \mathbf{P}_{j,i} [\mathbf{B}_i \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_i + \mathbf{B}_i \tilde{\mathbf{K}}_{r,j} \mathbf{r}_i] + \\ &2\gamma_j \mathbf{e}_i^T \mathbf{P}_{j,i} [\mathbf{B}_i (\mathbf{K}_{x,j}^{*T} - \mathbf{K}_{x,i}^{*T}) \mathbf{x}_i + \mathbf{B}_i (\mathbf{K}_{x,j}^* - \mathbf{K}_{x,i}^*) \mathbf{r}_i + \mathbf{B}_i (\mathbf{K}_{x,j}^T \mathbf{c}_{x,ji} + \mathbf{K}_{r,j} \mathbf{c}_{r,ji} + \mathbf{c}_{u,ji})] - \\ &2\gamma_j \mathbf{e}_i^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_j - 2\gamma_j \mathbf{e}_i^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{r}_j - 2\gamma_j (1 + k_3) (\gamma_j + k_3 \gamma_i) \rho \end{aligned} \quad (24)$$

Substituting the transformed variables $\mathbf{x}_j, \mathbf{r}_j, \mathbf{e}_j$ into Eq. (24), there is

$$\begin{aligned} \dot{V}_{j,i}(t) &\leq -\lambda_i \mathbf{e}_i^T \mathbf{P}_{j,i} \mathbf{e}_i - 2\gamma_j (1 + k_3) (\gamma_j + k_3 \gamma_i) \rho + 2\gamma_j \mathbf{e}_i^T \mathbf{P}_{j,i} [\mathbf{B}_i (\mathbf{K}_{x,j}^{*T} - \mathbf{K}_{x,i}^{*T}) \mathbf{x}_i + \mathbf{B}_i (\mathbf{K}_{x,j}^* - \mathbf{K}_{x,i}^*) \mathbf{r}_i] + \\ &2\gamma_j \mathbf{e}_i^T \mathbf{P}_{j,i} \mathbf{B}_i (\mathbf{K}_{x,j}^T \mathbf{c}_{x,ji} + \mathbf{K}_{r,j} \mathbf{c}_{r,ji} + \mathbf{c}_{u,ji}) - 2\gamma_j \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{e}_i - 2\gamma_j \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{e}_i - \\ &2\gamma_j \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_{m,i} - 2\gamma_j \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_{m,i} - 2\gamma_j \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{c}_{x,ji} - 2\gamma_j \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{c}_{x,ji} - \\ &2\gamma_j \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{r}_i - 2\gamma_j \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{r}_i - 2\gamma_j \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{c}_{r,ji} - 2\gamma_j \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{c}_{r,ji} \end{aligned} \quad (25)$$

With the assumption 1, make the following definitions:

$$\begin{aligned} a_{1,ji} &= \sup_{t \in \infty} \|\mathbf{P}_{j,i} [\mathbf{B}_i (\mathbf{K}_{x,j}^{*T} - \mathbf{K}_{x,i}^{*T}) \mathbf{x}_i + \mathbf{B}_i (\mathbf{K}_{x,j}^* - \mathbf{K}_{x,i}^*) \mathbf{r}_i]\| \\ a_{2,ji} &= \sup_{t \in \infty} \|\mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T - \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T\| \\ a_{3,ji} &= \sup_{t \in \infty} \|\mathbf{P}_{j,i} \mathbf{B}_i (\mathbf{K}_{x,j}^T \mathbf{c}_{x,ji} + \mathbf{K}_{r,j} \mathbf{c}_{r,ji} + \mathbf{c}_{u,ji})\| \\ a_{1,j} &= \sup_{t \in \infty} \left\| \begin{aligned} &-\mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_{m,i} - \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{x}_{m,i} - \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{c}_{x,ji} \\ &-\mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{x,j}^T \mathbf{c}_{x,ji} - \mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{r}_i - \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{r}_i \\ &-\mathbf{c}_{x,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{c}_{r,ji} - \mathbf{c}_{x,m,ji}^T \mathbf{P}_j \mathbf{B}_j \tilde{\mathbf{K}}_{r,j}^T \mathbf{c}_{r,ji} \end{aligned} \right\| \end{aligned}$$

Then Eq. (25) can be expressed as

$$\dot{V}_{j,i}(t) \leq -\lambda_{ji} \|\mathbf{P}_{j,i}\| \cdot \|\mathbf{e}_i\|^2 - 2\gamma_j (1 + k_3) (\gamma_j + k_3 \gamma_i) \rho + 2\gamma_j a_{1,ji} \|\mathbf{e}_i\| + 2\gamma_j a_{3,ji} \|\mathbf{e}_i\| + 2\gamma_j a_{2,ji} \|\mathbf{e}_i\| + 2\gamma_j a_{1,j} \quad (26)$$

As to the initial tracking error $\mathbf{e}_i(t_i)$ and given scalar $R_2 > 0$, the set $S(R_2) = \{\|\mathbf{e}\|^2 < R_2\}$ is defined. For any tracking error $\mathbf{e}_i(t) \in \mathbf{R}^n / S(R_2)$, there is

$$\dot{V}_{j,i}(t) \leq -\lambda_{ji} \beta_{ji, \min} \|\mathbf{e}_i\|^2 + 2\gamma_j \left[\frac{1}{R_2} (a_{1,ji} + a_{3,ji} + a_{2,ji}) \|\mathbf{e}_i\|^2 + a_{1,j} - (1 + k_3) (\gamma_j + k_3 \gamma_i) \rho \right] \quad (27)$$

Defining $d = \|\mathbf{e}_i\|^2 / R_2 (a_{1,ji} + a_{3,ji} + a_{2,ji}) + a_{1,j} - (1+k_3)(\gamma_j + k_3\gamma_i)\rho$, then $\dot{V}_{j,i}(t) \leq -\lambda_{ji} \cdot \beta_{ji,\min} \|\mathbf{e}_i\|^2 + 2\gamma_j d$ from which it follows that $\dot{V}_{j,i}(t) \leq 0$ when $1/\rho \cdot R_2 (a_{1,ji} + a_{3,ji} + a_{2,ji}) \|\mathbf{e}_i\|^2 + \alpha_j \leq k_3$.

Thus, all signals in the closed-loop system are bounded and the asymptotic convergence of $\tilde{\mathbf{K}}_{x,i}^T, \mathbf{K}_{r,i}$ and $\mathbf{e}_i(t)$ can be ensured. It can be concluded, with the inequality, that there exists $0 < k_3 \in \mathbf{R}^+$ for any $\mathbf{e}_i$ to guarantee the stability of the tracking error dynamic system Eq. (17).

Combining Eq. (20), Eq. (27) and the above mentioned k_3 , there is

$$\dot{V}_{j,i}(t) \leq -\frac{\lambda_{ji}\beta_{ji,\min}}{\beta_{ji,\max}(1+k)}V_{j,i}(t) + \frac{(1+k)\rho - k\lambda_{ji}\beta_{ji,\min}V_{j,i}(t)}{\beta_{ji,\max}(1+k)} \quad (28)$$

where $k > 0$. For $V_{j,i}(t) \geq (1+k)\frac{1}{k}\rho$, $V_{j,i}(t)$ decays faster than exponentially at the rate $\frac{\lambda_{ji}\beta_{ji,\min}}{(1+k)\beta_{ji,\max}}$. Therefore, the $V_{j,i}(t)$ can be guaranteed not to increase, i. e., $V_{j,i}(t) \leq V_{j,i}(t_i)$, $t \in [t_i, t_i + \tau(t_i))$.

The global stability and tracking characteristics of the piecewise closed-loop system can be summarized in the following theorem.

Theorem 2 Considering the closed-loop system contains reference model (Eq. (7)), piecewise linear system (Eq. (5)), smooth switching scheme (Eq. (6)), and state feedback control law (Eq. (12)) with adaptive law Eq. (20). Let the reference signals $\mathbf{r}_i$ be sufficiently rich of order $(n+1)$ with distinct frequencies. If the reference model $(\mathbf{A}_{m,i}, \mathbf{B}_{m,i})$ are controllable and the switching signal be sufficiently slow with dwell time T_{dwell} , then the estimation error $\tilde{\mathbf{K}}_{x,i}^T, \tilde{\mathbf{K}}_{r,i}$ and tracking error asymptotically converge to zero for $t \rightarrow \infty$

Proof When $t \in [t_i + \tau(t_i), t_{i+1})$, $\chi_i(t) = 1$, $\chi_j(t) = 0$ and the tracking error dynamic Eq. (15) can be rewritten as

$$\dot{\mathbf{e}}_i = \mathbf{A}_{m,i}\mathbf{e}_i + \mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}[\tilde{\mathbf{K}}_{x,i}^T \tilde{\mathbf{K}}_{r,i}] \begin{bmatrix} \mathbf{x}_i \\ \mathbf{r}_i \end{bmatrix} \quad (29)$$

Make use of the Kronecker product $\otimes$ and

define

$$\Psi_r = \begin{bmatrix} \mathbf{x}_i \\ \mathbf{r}_i \end{bmatrix} \otimes \mathbf{I}_n$$

$$\tilde{\mathcal{J}}_i^B = \text{vec}(\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}[\tilde{\mathbf{K}}_{x,i}^T \tilde{\mathbf{K}}_{r,i}])$$

Where the operator $\text{vec}(\cdot)$ denotes the vectorization of the given matrix formed by stacking the columns of the matrix into a vector. With the vectorization of matrix and Kronecker product, there is

$$\dot{\mathbf{e}}_i = \mathbf{A}_{m,i}\mathbf{e}_i + \Psi_r^T \tilde{\mathcal{J}}_i^B \quad (30)$$

The time derivative of $\tilde{\mathcal{J}}_i^B$ can be given by

$$\dot{\tilde{\mathcal{J}}}_i^B = \text{vec}(\mathbf{B}_i[\dot{\tilde{\mathbf{K}}}_{x,i}^T \dot{\tilde{\mathbf{K}}}_{r,i}]) \quad (31)$$

In view of the update laws (Eq. (19)) and various steps of algebraic reformulation, the derivative of $\tilde{\mathcal{J}}_i^B$ can be written as

$$\begin{aligned} \dot{\tilde{\mathcal{J}}}_i^B &= \text{vec}(\mathbf{B}_i[\dot{\tilde{\mathbf{K}}}_{x,i}^T \dot{\tilde{\mathbf{K}}}_{r,i}]) = \\ &= -\text{vec}(\mathbf{B}_{m,i}\mathbf{K}_{r,i}^{*-1}[\mathbf{S}_i^T \mathbf{B}_{m,i}^T \mathbf{P}_i \mathbf{e}_i \mathbf{x}_i^T \quad \mathbf{S}_i^T \mathbf{B}_{m,i}^T \mathbf{P}_i \mathbf{e}_i \mathbf{r}_i^T]) = \\ &= -\Psi_r \mathbf{B}_{m,i} \mathbf{K}_{r,i}^{*-1} \mathbf{S}_i^T \mathbf{B}_{m,i}^T \mathbf{P}_i \mathbf{e}_i \end{aligned} \quad (32)$$

Combining Eq. (29) and Eq. (30), there is

$$\dot{\mathbf{z}}_i = \mathbf{A}_{a,i} \cdot \mathbf{z}_i \quad (33)$$

Where

$$\mathbf{A}_{a,i} = \begin{bmatrix} \mathbf{A}_{m,i} & \Psi_r^T \\ -\Psi_r \mathbf{B}_{m,i} \mathbf{K}_{r,i}^{*-1} \mathbf{S}_i^T \mathbf{B}_{m,i}^T \mathbf{P}_i & \mathbf{0} \end{bmatrix}, \mathbf{z}_i = \begin{bmatrix} \mathbf{e}_i \\ \tilde{\mathcal{J}}_i^B \end{bmatrix}$$

And the Eq. (33) is a standard form in system stability analysis and adaptive systems.

Exponential stability of system (Eq. (33)) can be ensured if the augmented system state $\mathbf{z}_i$ is persistent excitation^[21-22], and the detailed proof can be seen in the Ref. [22]. As the system $(\mathbf{A}_{m,i}, \mathbf{B}_{m,i})$ is controllable and the reference signal $\mathbf{r}_i$ is persistent excitation, then it can be concluded that the state $\mathbf{x}_{m,i}$ of reference system and the state $\mathbf{x}_i$ of controlled system are persistent excitation. Then, there is

$$\|\mathbf{z}_i(t)\| \leq a_i e^{-\lambda(t-t_i)} \cdot \|\mathbf{z}_i(t_i)\| \quad (34)$$

When $t \in [t_i + \tau(t_i), t_{i+1})$, the decrease of Lyapunov function (Eq. (18)) can be written as

$$V_i(t_{i+1}) \leq \frac{\beta_{i,\max}}{\beta_{i,\min}} \cdot a_i^2 e^{-2\lambda_i[t_{i+1} - (t_i + \tau(t_i))]} \cdot V_{j,i}(t_i)$$

To form a descending sequence of Lyapunov function, the time interval T_i satisfies

$$T_i = t_{i+1} - (t_i + \tau(t_i)) \geq \frac{1}{2\lambda_i} \ln \mu_e \cdot \frac{\beta_{i,\max}}{\beta_{i,\min}} a_i^2$$

Which defines a time interval T_i similar to the dwell time for the i -th subsystem. The global stability and convergence of the tracking errors and estimated control gains can be ensured if the switching of the controlled system obeys the time interval $T_d > \max\{T_1, T_2, \dots, T_k, \dots\}$, $0 \leq k \in \mathbf{Z}$. This means that $t_{i+1} - (t_i + \tau(t_i)) \leq T_d, i \in I$, and a time constraint is set for switching signals.

4 Simulation validation

The following simulation illustrates the effectiveness of MRAC scheme with smooth indicator functions for the system in comparison with MRAC scheme with direct switching. In this section, we consider a turbofan engine similar to the GE (General Electric) CF6 series, which is modeled in Matlab/Simulink, and the maximum fan speed $N_1 = 3\,390$ r/min and core speed $N_2 = 10\,300$ r/min^[23-24]. The nonlinear engine model is a component level iterative model, and all engine performance maps and component parameters are acquired from gas turbine simulation program (GSP). N_1 and N_2 are selected as the state variables, and the fuel flow W_f is regarded as the control input. The thrust is controlled indirectly by controlling N_1 according to the turbofan engine control practice, and N_1 is selected as desired output variable.

Piecewise linear models are acquired based on the linearized model described in section 2, and then MRAC controllers and adaptive updating laws are designed. The reference models used for generating desired state trajectories are chosen to be the system with poles placed at -3 and -5 . We select some working points in the flight envelope and linearize the system at these working points. For simplicity of simulation verification, 4 steady operating points are selected, and these steady operating points are selected to describe the large-scale engine operation from idle to full power for the condition $H=0$ m and $Ma=0$. The initial value of the adaptive varia-

bles $\mathbf{K}_{r,i}, \mathbf{K}_{x,i}, i=1,2,3,4$ are set as $\mathbf{K}_{r,i}(0)=5 \times 10^{-5}$, $\mathbf{K}_{x,i}(0)=\text{diag}[3.2 \times 10^{-4}, 3.2 \times 10^{-4}]$, respectively. The designed parameters are selected as $c_4=8, k_4=18$, $\mathbf{F}_{x,i}=\text{diag}[1 \times 10^{-6}, 1 \times 10^{-6}]$, $\mathbf{F}_{r,i}=1 \times 10^{-4}$.

All controller parameters are suitably retuned to get preferable dynamic performance, and limit protection problem hasn't been considered in the simulation. The dynamic response of closed-loop subsystem with well retuned parameters can be seen in Fig. 2, and the corresponding control input is illustrated in Fig. 3. In Fig. 4 and Fig. 5, the term IF is the acronym for indicator function, and the gray lines denote the

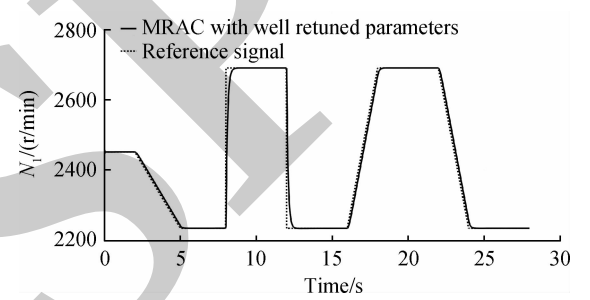


Fig. 2 Dynamic response of subsystem with well retuned MRAC parameters

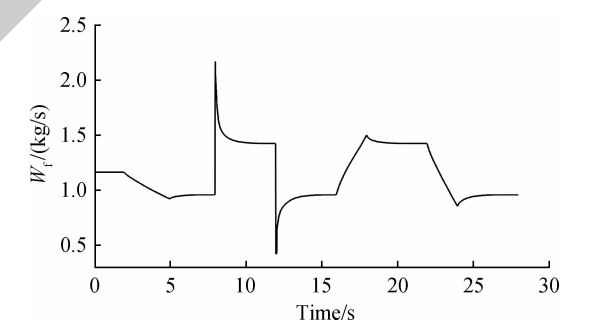


Fig. 3 Fuel flow of the subsystem with well retuned MRAC parameters

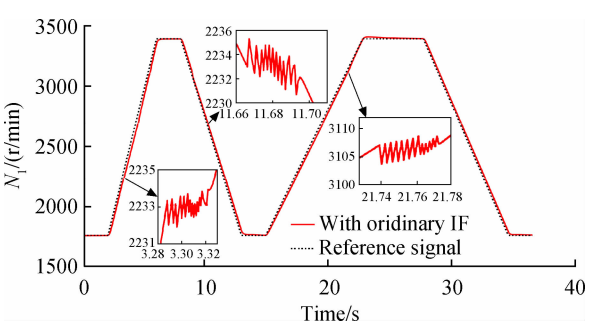


Fig. 4 N_1 response of MRAC with conventional indicator functions during the operation

boundary of two neighboring sub-regions and system state switches at the boundary.

In the following, simulations are performed for two cases with the same reference signal as shown in Fig. 4 and Fig. 5. As can be seen in Fig. 4, the closed-loop system consisting of MRAC with conventional indicator functions can track the desired trajectories. However, the chattering occurs when switching from one sub-region to another and the system output chattering is shown in the partial enlarged view. The history of indicator functions $\chi_i(t), i=1,2,3,4$ are presented in Fig. 6.

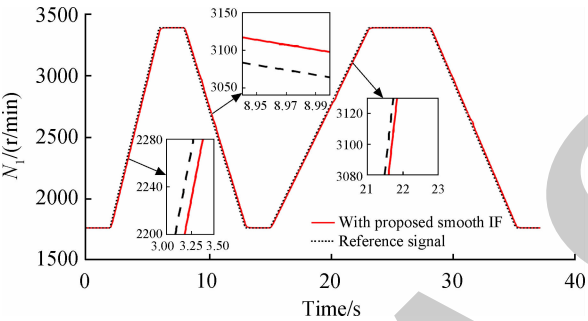


Fig. 5 N_1 response of MRAC with smooth indicator functions during the operation

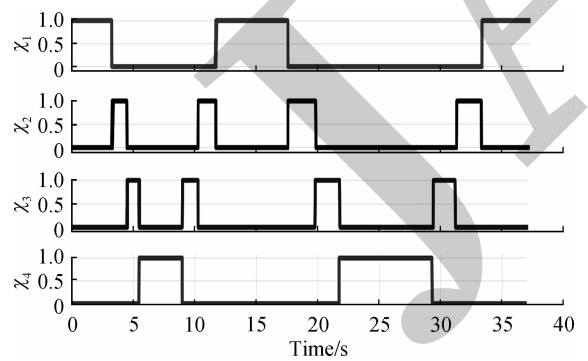


Fig. 6 Conventional indicator function of each subsystem

Fig. 5 and Fig. 7 show the tracking performance of MRAC with proposed smooth switching scheme. As can be seen, the turbofan engine accelerates from idle to maximum speed in 4.2 s, and the MRAC with smooth indicator functions yield desired performance during the operation. The system output during the switch process is much smoother, and chattering is avoided. With the proposed smooth switching scheme, the instability may be caused by asynchronous switch-

ing can be avoided as the controller switching isn't accomplished at certain instant of time. The fuel flow of MRAC with two different kinds of indicator functions are presented in Fig. 8. It can be seen that the fuel flow of MRAC with conventional indicator functions experience larger swings or chattering.

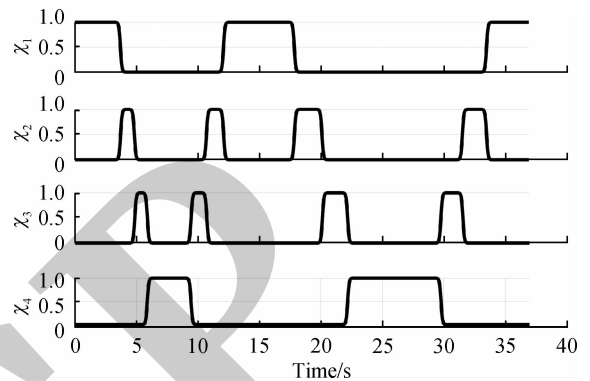


Fig. 7 Smooth indicator function of each subsystem during the operation

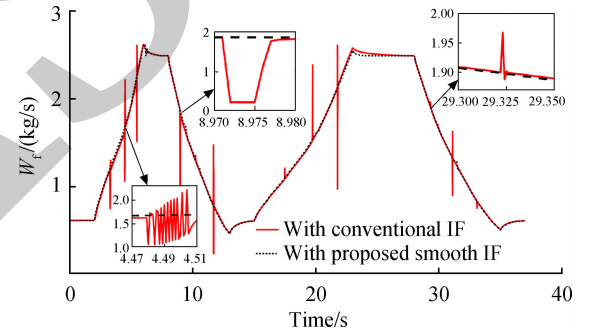


Fig. 8 Fuel flow during the operation

5 Conclusion

This paper proposes a smooth switching scheme based MRAC controller for aircraft engine to avoid chattering of system output and discontinuity of control input during the switching process. And the instability may be caused by asynchronous switching is also eliminated. Due to the dynamic characteristics of engine vary greatly in the whole working process, only one MRAC controller can't achieve desired dynamic performance and ensure global stability within the operating envelope. To ensure the smooth switching of multiple controllers during the operation of engine, the parameters of the induced

smooth indicator functions should satisfy certain conditions. The simulation results exhibit the good tracking performance and indicate the effectiveness of the proposed control scheme. In order to ensure the global stability and the convergence of parameter errors and tracking error, Lyapunov functions are considered and dwell-time constraint on the switching signal is give. In the future study, other switching schemes suitable for MRAC and more appropriate models which can approximate the dynamic characteristics of engine can be considered.

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